It should be emphasized that in both the simple and multiple-pole cases, the asymptotic behavior deduced puts an upper bound on the behavior; the residues may vanish more rapidly.

IV. CONCLUSION

We have established the conditions under which Eqs. (II.1) and (II.2) are expected to comprise a good approximation to the scattering amplitude in the strip region of Fig. 1. The most important requirement for the consistency of the approximation is that the residues β vanish at infinity. As we have indicated in footnote 5, it is not necessary to require that the residues fall off as rapidly as the inverse square root of the energy in order for the neglected part of the amplitude to vanish at infinity. It should be mentioned, however, that if the vanishing of the residues is weaker than inverse square-root behavior, we shall generally find spurious singularities to the right of $\mathrm{Re}\, l=\frac{1}{2}$ and we may not be able to satisfy property (3) of Sec. II.

An examination of the dynamical equations in the asymptotic limit has shown that the behavior of $\beta(s)$ at infinity is controlled by $\alpha(\infty)$, a quantity not known precisely before numerical calculations are performed. We can, however, conclude in the case of the Fredholm pole giving rise to $\alpha(\infty)$ is simple that the restriction of $\alpha(\infty)$ to less than unity guarantees the asymptotic vanishing of $\beta(s)$. Thus there is every indication that self-consistent solutions to the strip equations will exist although a detailed verification of self-consistency must await numerical solution of the integral equations.

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Possible Existence of a Boson Icosuplet

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It is pointed out that the resonance (B_1) of the $\pi\omega$ system at 1220 MeV and the resonance (B_2) of the $\pi\rho$ system at 1200 MeV may form parts of an icosuplet consisting of a boson decuplet and its charge conjugate. Mixing of the two isotriplets in the decuplet and the conjugate decuplet is noted: B_1 and B_2 which are eigenstates of the operator G are linear combinations of isotriplets in the $\{10\}$ and $\{10\}^*$ representations. Consequences of this assumption are explored. The possibility that the $K\pi\pi$ resonance at 1175 MeV may belong to this icosuplet is also discussed.

RECENTLY, a resonance in the $\pi\omega$ system (B_1) has been found at 1220 MeV.[1] Furthermore, there are some indications that a $\pi\rho$ resonance exists at 1200 MeV.[2] The isotopic spin of the $\pi\rho$ resonance is known to be greater than zero[2]; in the present note we shall assume it to be 1. The purpose of this note is to point out the possibility that they may form parts of an icosuplet[3] made up of a boson decuplet and its charge conjugate in the unitary symmetry model[4] of strong interactions, and to investigate the consequences of this assumption. We shall also investigate the possibility that the newly discovered $\pi\pi K$ resonance at[5] 1175 MeV may be a part of this boson multiplet.

There are two interesting features of a boson decuplet and its antiparticle multiplet:

(1) There are two physically observable isotriplets of zero strangeness. They are two linear combinations of the triplets in the decuplet representation and its conjugate.

(2) If the spin-parity of the icosuplet is 0^+, 2^+, 4^+, etc., particles of this multiplet cannot decay into two pseudoscalar octet mesons in the limit of unitary symmetry. (If the spin-parity is 1^+, 3^+,$\cdots$ the decay is for-

[1] M. Abolins, R. L. Lander, W. W. Mehlhop, Ng.h. Xuong, and P. M. Yager, Phys. Rev. Letters **11**, 381 (1963).
[2] G. Goldhaber, J. Brown, S. Goldhaber, J. Kadyk, B. Shen, and G. Trilling (to be published).
[3] From the Greek prefix "εικοσι" as in icosahedron. The equivalent Latin form is "*viginti*." The etymologically impure (since the suffix "*plet*" seems to originate from Latin "*plex*") form "icosuplet" is preferred here over "vigintuplet" because of ease in pronunciation.
[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); unpublished. Y. Neéman, Nucl. Phys. **26**, 222 (1962).

[5] T. P. Wangler, W. D. Walker, and A. R. Erwin (to be published).

TABLE I. Relative decay widths (for $l=0,1,2$) of the $Q=-1$ members of the proposed icosuplet. Estimates I and II are based on Eqs. (1) and (2), respectively.

	Decay mode	Relative weight	Mass (in MeV)	Relative width (I) s	p	d	Mass (in MeV)	Relative width (II) s	p	d
$S=1$	$K^{*0}+\pi^-$	1	1175	1.237	0.960	0.464	1265	1.552	2.193	1.934
$T=\frac{3}{2}$	ρ^-+K^0	1		...	...	...		0.557	0.103	0.011
$S=0$	$\omega+\pi^-$	$\sin^2\theta$	1220	0.460	0.847	0.973				
$T=1$	$\phi+\pi^-$	$\cos^2\theta$		0.540	0.153	0.027				
$G=1$	$\rho^-+\eta$	1		...	...	...				
$S=0$	$\rho^-+\pi^0$	$\frac{1}{3}$	1200	0.635	1.218	1.457				
$T=1$	$\rho^0+\pi^-$	$\frac{1}{3}$		0.635	1.218	1.457				
$G=-1$	$\bar{K}^{*-}+K^0$	$\frac{2}{3}$		...	...	...				
	$\bar{K}^{*0}+K^-$	$\frac{2}{3}$		...	...	...				
$S=-1$	ρ^0+K^-	$\frac{1}{6}$	1265	0.093	0.017	0.002	1175	...	...	...
$T=\frac{1}{2}$	$\rho^-+\bar{K}^0$	$\frac{1}{3}$		0.186	0.034	0.004		...	...	...
	$\omega+K^-$	$\frac{1}{2}\sin^2\theta$		...	...	...		...	...	...
	$\phi+K^-$	$\frac{1}{2}\cos^2\theta$		...	...	...		...	...	...
	$\bar{K}^{*0}+\pi^-$	$\frac{1}{3}$		0.517	0.732	0.645		0.412	0.320	0.155
	$\bar{K}^{*-}+\pi^0$	$\frac{1}{6}$		0.259	0.366	0.323		0.206	0.160	0.078
	$\bar{K}^{*-}+\eta$	$\frac{1}{2}$		...	...	...		...	...	...
$S=-2$	$\bar{K}^{*0}+\bar{K}^-$	1	1310	...	...	...	1130	...	...	...
$T=0$	$\bar{K}^{*-}+\bar{K}^0$	1		...	...	...		...	...	...

bidden by conservation of angular momentum and parity, of course.)

A boson decuplet and its charge conjugate, which we shall designate simply by $\{10\}$ and $\{10\}^*$, contain the following isospin multiplets:

$$\{10\}: \quad (I=\tfrac{3}{2},\ S=1),\ (I=1,\ S=0),\ (I=\tfrac{1}{2},\ S=-1),\ (I=0,\ S=-2).$$

$$\{10\}^*: \quad (I=\tfrac{3}{2},\ S=-1),\ (I=1,\ S=0),\ (I=\tfrac{1}{2},\ S=1),\ (I=0,\ S=2).$$

There are two isotriplets with zero strangeness, one each in $\{10\}$ and $\{10\}^*$. Neither of them has a definite G parity. Since the unitary symmetry is broken in nature, physically observed triplets are eigenstates of the operator G. The situation here is very similar to the particle mixing in weak interactions. There we have K_1^0 and K_2^0, which are eigenstates of CP and are linear combinations of K^0 and $\bar{K}^0$, as particles with definite masses and widths (and decay modes). Let Ψ be any member of the isotriplet belonging to $\{10\}$. We form

$$B_1=(1/\sqrt{2})(\Psi+G\Psi),$$
$$B_2=(1/\sqrt{2}i)(\Psi-G\Psi),$$

where G is the G conjugation operator. Obviously,

$$GB_1=+B_1,$$
$$GB_2=-B_2.$$

Physically, it is B_1 and B_2 rather than Ψ and $G\Psi$ that have definite masses and lifetimes. We may identify B_1 with the $\pi\omega$ resonance (1220 MeV) and B_2 with the $\pi\rho$ resonance (1200). This identification has the attractive feature that it can explain rather naturally the near mass degeneracy of B_1 and B_2. If we assume as usual the octet behavior of mass splitting interactions,[6] there is no first order mass difference between B_1 and B_2 in perturbation theory. Hence it is only in the second order and higher that we expect to find the mass difference between B_1 and B_2. The equal mass splitting relation among decuplet particles[6,7] is unaffected by $\Psi-G\Psi$ mixing in first-order perturbation theory.

The coupling of two octets to a decuplet ($\{10\}$ or $\{10\}^*$) is antisymmetric in the two octet indices. Therefore, in the limit of exact symmetry, a decuplet meson can decay into two pseudoscalar mesons of the same octet only if the spin-parity of the decuplet is 1^-, 3^-, etc. Experimentally, decays of B_1 and the $\pi\pi K$ resonance into two pseudoscalars have not been seen. This can be explained, therefore, if they belong to an icosuplet, even if their spin-parity turns out to be 2^+, 4^+, etc. (it cannot be 0^+, since, then, B_1 cannot decay into $\pi+\omega$).

If the spin-parity of B_2 should turn out to be 1^-, B_2 could decay into two pseudoscalar octets. Indeed if we observe the decay into $\pi\eta$ in a p state (or more generally in any odd angular momentum state), we must conclude that B_2 belongs to an icosuplet. It cannot belong to either the $\{8\}$ or $\{27\}$ representation as can be shown by a theorem due to Lipkin.[8] This can be understood also as follows: suppose that we have a $\pi\eta$ resonance in an odd angular momentum state. Then, because of Bose statistics, the two bosons must be in an

[6] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[7] See for instance S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).
[8] H. Lipkin (unpublished).

antisymmetric charge state. This fact eliminates the {27} representation and the symmetric octet representation. We can, however, form an antisymmetric octet combination, the so-called F coupling. We can easily show, though, that this representation does not contain the $\pi\eta$ system. Therefore, a $\pi\eta$ resonance in an odd parity state must be in either {10} or {10}* or their linear combination. Actually a bootstrap calculation seems to indicate the existence of such a p-wave $\pi\eta$ resonance,[9] but the details will be published elsewhere.

We have computed relative decay rates of B_1 and B_2 into a pseudoscalar meson and a vector meson (see Table I). Relative widths are computed for three different values (0,1,2) of the final state orbital angular momentum l, taking into account the phase space-centrifugal barrier factor k^{2l+1}/m, where k is the decay momentum and m is the resonance mass. For decays of B_1 into $\omega+\pi$ and $\phi+\pi$ we have considered the effect of $\omega-\phi$ mixing.[10] The physical ω and ϕ are related to an octet ω_8 and a singlet ω_1 through

$$\begin{pmatrix}\omega\\ \phi\end{pmatrix}=\begin{pmatrix}\cos\theta, & \sin\theta\\ -\sin\theta, & \cos\theta\end{pmatrix}\begin{pmatrix}\omega_1\\ \omega_8\end{pmatrix}.$$

It is the ω_8 components of ω and ϕ that contribute to the decay of B_1. We have taken $\theta=30°$ (Sakurai[10] and Okubo[10] deduced an angle of $\simeq 33°$. A recent study by Kim and Oneda[11] indicate a smaller angle, about 25°). Assuming the dominant decay of the icosuplet to be into a pseudoscalar meson and a vector boson, we have computed the total width of B_2 [$\Gamma(B_1)\simeq 100$ MeV is assumed]:

	Γ_s	Γ_p	Γ_d
$B_1(G=+1)$:	100 MeV	100	100
$B_2(G=-1)$:	127	244	291

In interpreting the above result we must bear in mind that the ρ meson has a large width of about 100 MeV, and the real width of the B_2 is the natural width modified by the width of the ρ meson. Present meager experimental data appear to be compatible with the above estimate for $l=1$ (spin-parity $0^-,1^-,2^-$) or $2(1^+,2^+,3^+)$.

Let us assume, provisionally, that the $\pi\pi K$ resonance (1175 MeV) belongs to the same icosuplet as B_1 and B_2.[12] If it has $I=\frac{3}{2}$, the Gell-Mann[4] Okubo[6] mass formula gives

$$\begin{aligned} M(I=\tfrac{3}{2})&=1175\text{ MeV},\\ M(I=1)&=1220\text{ MeV},\\ M(I=\tfrac{1}{2})&=1265\text{ MeV},\\ M(I=0)&=1310\text{ MeV}.\end{aligned}\tag{1}$$

If the isospin of the $\pi\pi K$ resonance (1175) is $\frac{1}{2}$, we predict

$$\begin{aligned} M(I=\tfrac{3}{2})&=1265\text{ MeV},\\ M(I=1)&=1220\text{ MeV},\\ M(I=\tfrac{1}{2})&=1175\text{ MeV},\\ M(I=0)&=1130\text{ MeV}.\end{aligned}\tag{2}$$

Note that in both cases we should have another $\pi\pi K$ resonance at 1265 MeV. Based on the predictions (1) and (2), we have computed relative decay rates of members of the decuplet into a pseudoscalar meson and a vector boson. The results are shown in Table I. The predicted resonance at 1265 MeV has a rather large width (see Table I) and should have been seen in the experiment of Wangler *et al.*[5] It is therefore unlikely, on the basis of *existing data*, that the $\pi\pi K$ resonance belongs to the icosuplet. Note that if the prediction (1) or (2) is correct, the $I=0$, $S=-2$ particle cannot decay into $\bar{K}^*+\bar{K}$ and decays instead into $\bar{K}+\bar{K}+\pi$.

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[9] J. Schecter and S. Okubo (to be published).

[10] J. J. Sakurai, Phys. Rev. **132**, 434 (1963). S. Okubo, Phys. Letters **5**, 165 (1963). S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963). J. Kalckar, Phys. Rev. **131**, 2242 (1963).

[11] Y. S. Kim and S. Oneda (to be published).

[12] A different theoretical interpretation is given by C. Goebel (to be published). The fact that there is no evidence for decay of $K\pi\pi(1175)$ into a $K^*\pi$ system beyond what one would expect for a mass of 1175 MeV and a simple phase-space distribution is not necessarily an evidence against our provisional assignment, since on the basis of decay widths, $\Gamma(\pi\pi K)<50$ MeV and $\Gamma(K^*)\simeq 50$ MeV, we expect K^* to decay into $K+\pi$ before K^* can leave the interaction volume. In this case we expect the $K\pi$ spectrum to be distorted by the final state $\pi\pi$ interactions.